The Georgi "Avatar" of Broken Chiral Symmetry in Quantum Chromodynamics

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Abstract

We establish that in Quantum Chromodynamics (QCD) at zero temperature, $SU_{L+R}(N_F)$ exhibits the vector mode conjectured by Georgi and $SU_{L-R}(N_F)$ is realized in either the Nambu-Goldstone mode or else Q_5^a is also screened from view at infinity. The Wigner-Weyl mode is ruled out unless the beta function in QCD develops an infrared stable zero.

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We shall establish that QCD at zero temperature, satisfying both asymptotic freedom and confinement, exhibits the following features: (a) $SU_{L+R}(N_F)$ exhibits the vector mode conjectured by Georgi [1] (Georgi-Goldstone mode). (b) $SU_{L-R}(N_F)$ exhibits either the Nambu-Goldstone mode or else the axial-vector charge Q_5^a is also screened from view at infinity. If the latter case were to occur, then QCD confines without breaking chiral symmetry: both $SU_{L+R}(N_F)$ and $SU_{L-R}(N_F)$are realized in the Higgs mode (Georgi-Wigner mode), with no scalar or pseudoscalar Nambu-Goldstone bosons and the vector and axial-vector mesons are degenerate. (c) The Wigner-Weyl mode corresponding to $Q^a \mid 0 \rangle = 0$, $Q_5^a \mid 0 \rangle = 0$ is ruled out: the Callan-Symanzik beta function has to turn over to yield an infrared stable fixed point at a finite value of g if chiral symmetry is to be restored.

We now sketch the proof of these interesting assertions. We begin with the vector current V_μ^a and its conservation

$$\partial^\mu V_\mu^a (\mathbf{x}, t) = 0. \tag{1}$$

This implies the local version

$$[Q^a (t), H (\mathbf{x}, t)] = 0 \tag{2}$$

where $H (\mathbf{x}, t) = \Theta^{00}$ is the Hamiltonian density, if the surface terms at infinity can be discarded. This is clearly justified if the flavor vector charge annihilates the vacuum,

$$Q^a(t) \mid 0 \rangle = 0 \tag{3}$$

which is guaranteed by the Vafa-Witten theorem [2,3] *i.e.,* non-chiral symmetries cannot be spontaneously broken in vector-like gauge theory. Hence there are no scalar Nambu-Goldstone bosons to produce a long range interaction, which in turn would have resulted in a non-vanishing contribution to the surface terms.

The dilatation charge

$$Q_D(t) = \int d^3x D_0 (\mathbf{x}, t), \tag{4}$$

defined in terms of the dilatation current $D_\mu (\mathbf{x}, t)$ satisfies [4] the trace anomaly

$$\partial^\mu D_\mu = \frac{\beta(g)}{2g} G_{\mu\nu}^\alpha G_\alpha^{\mu\nu} \tag{5}$$

in QCD in the chiral limit when the current quark mass is zero. It is well-known that scale invariance is broken both "spontaneously", $Q_D(t) \mid 0 \rangle \neq 0$, and explicitly by the trace anomaly. Consequently, the states defined by successive repeated application of $Q_D(t)$ on the vacuum state are neither vacuum states nor are they necessarily degenerate [5]. Let the commutator

$$[Q_D(0), Q^a(0)] = -id_Q Q^a(0) \tag{6}$$

define the scale dimension d_Q of the charge $Q^a(0)$. By translation invariance, this can be put in the form

$$[Q_D(t), Q_a(t)] = -i d_Q Q_a(t). \tag{7}$$

It is important to stress that operator relations such as the above equation are unaffected by spontaneous symmetry breaking as emphasized by Weinberg [6]. Let us consider the double commutator which follows from Eq. (2),

$$[Q_D(t), [Q_a(t), H]] = 0, \tag{8}$$

where $Q_D(t)$ is the dilation charge defined in Eq. (4). If we now invoke the Jacobi identity we can recast the above equation in the form

$$[Q_a(t), [H, Q_D(t)]] + [H, [Q_D(t), Q_a(t)]] = 0. \tag{9}$$

Since

$$[H(\mathbf{x}, t), Q_D(t)] = -i\partial_\mu D^\mu(\mathbf{x}, t) \neq 0, \tag{10}$$

by virtue of the trace anomaly, Eq. (5), and making use of Eqs. (2,7), we arrive at the operator relation

$$[Q_a(t), \partial^\mu D_\mu(\mathbf{x}, t)] = 0. \tag{11}$$

Applying this relation on the vacuum state, we obtain

$$[Q_a(t), \partial^\mu D_\mu(\mathbf{x}, t)] \,|\, 0 \,\rangle \; = 0. \tag{12}$$

We may now invoke the result of Vafa-Witten theorem [2]

$$Q_a(t) \,|\, 0 \,\rangle \; = 0, \tag{13}$$

and conclude that

$$\mathcal{O}(\mathbf{x}, t) \,|\, 0 \,\rangle \; \equiv Q_a(t)\partial^\mu D_\mu(\mathbf{x}, t) \,|\, 0 \,\rangle \; = 0, \tag{14}$$

where the operator $\mathcal{O}(\mathbf{x}, t)$ is *local* in space and time and commutes with itself for space-like intervals. Explicitly, we see that this condition reduces to

$$[Q^a(t)\partial^\mu D_\mu(\mathbf{x}, t), Q^a(t')\partial^\mu D_\mu(\mathbf{x}', t')] = Q^a(t)Q^a(t')\,[\partial^\mu D_\mu(\mathbf{x}, t), \partial^\mu D_\mu(\mathbf{x}', t')] = 0 \tag{15}$$

for space-like intervals, due to the time independence of $Q^a(t)$ and Eq. (11). Here we have invoked the locality of $\partial^\mu D_\mu(\mathbf{x}, t)$.

We can now utilize the Federbush-Johnson theorem [7], which applies to any local operator, to Eq. (14) and immediately arrive at the key result,

$$Q_a(t)\partial^\mu D_\mu(\mathbf{x}, t) \equiv 0. \tag{16}$$

Since $\partial^\mu D_\mu(\mathbf{x}, t)$ cannot vanish in a theory which exhibits both asymptotic freedom and confinement except at $g = 0$, we conclude that the vector flavor charges must be screened from view at infinity,

$$Q^a(t) = 0. \tag{17}$$

This important result is a manifestation of spontaneously broken local symmetry. The vector mesons become massive and the scalar would-be Nambu-Goldstone bosons disappear.

Let us now consider the axial-vector charges Q_5^a. The Vafa-Witten theorem does not apply in this case and therefore we proceed by the method of *reductio ad absurdum* as follows. We assume $Q_5^a \,|\, 0 \,\rangle \;=\; 0$ corresponding to the Wigner-Weyl mode of unbroken symmetry. We begin by defining the scale dimension of the axial-vector charge by

$$[Q_D(0), Q_5^a(0)] = -id_{Q_5} Q_5^a(0). \tag{18}$$

Repeating the earlier analysis now for the axial-vector charges, exactly as in Eqs. (7–15), we arrive at the conclusion

$$Q_5^a(t)\partial^\mu D_\mu(\mathbf{x}, t) \equiv 0. \tag{19}$$

Since $Q_5^a(t) \neq 0$ for the assumed Wigner-Weyl mode, this requires $\partial^\mu D_\mu(\mathbf{x}, t) = 0$. This would imply that QCD exhibiting both asymptotic freedom and confinement is free. Hence by *reductio ad absurdum* we are led to the conclusion: either $Q_5^a(t) \,|\, 0 \,\rangle \neq 0$ or $Q_5^a(t) \equiv 0$. The first alternative is the Nambu-Goldstone realization of chiral symmetry which must hold for $N_F = 3$, [6]. The second alternative in conjunction with the screening of the vector charges, *i.e.*, $Q^a(t) \equiv 0$, $Q_5^a(t) \equiv 0$, is the Higgs mode alternative (Georgi-Wigner mode): both scalar and pseudoscalar Nambu-Goldstone bosons have been devoured. This case corresponds to confinement with exact chiral symmetry. Such a mode is realized in Supersymmetric QCD [8].

In conclusion, it is interesting that the Wigner-Weyl mode is ruled out in QCD at $T = 0$, if both asymptotic freedom and confinement obtains. this follows from Eqs. (16,19). Hence the Wigner-Weyl mode can occur only if the beta function turns over to yield an infrared stable fixed point [9]. Which of the above alternative realizations indeed occurs in QCD cannot be settled by this analysis. The answer may depend on the number of flavors.

An effective Lagrangian [10,11] which realizes the Georgi-Goldstone mode is easily constructed, following the genral procedure for building models with vector and axial vector mesons.

Finally, it is important to stress that we have not invoke the $SU_L(N_F) \times SU_R(N_F)$ charge algebra in our analysis. In view of the screened charges, *i.e.*, $Q^a(t) = 0$ for the Georgi-Goldstone mode and $Q^a(t) = Q_5^a(t) = 0$ for the Georgi-Wigner mode, one must revert back to current algebra [6] which leads to the Weinberg sum rules in QCD.

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